



Alignment in bioinformatics..Gene and gene regulatory networks

Biology (University of Ilorin)

Lecture 2

Sequence Alignment

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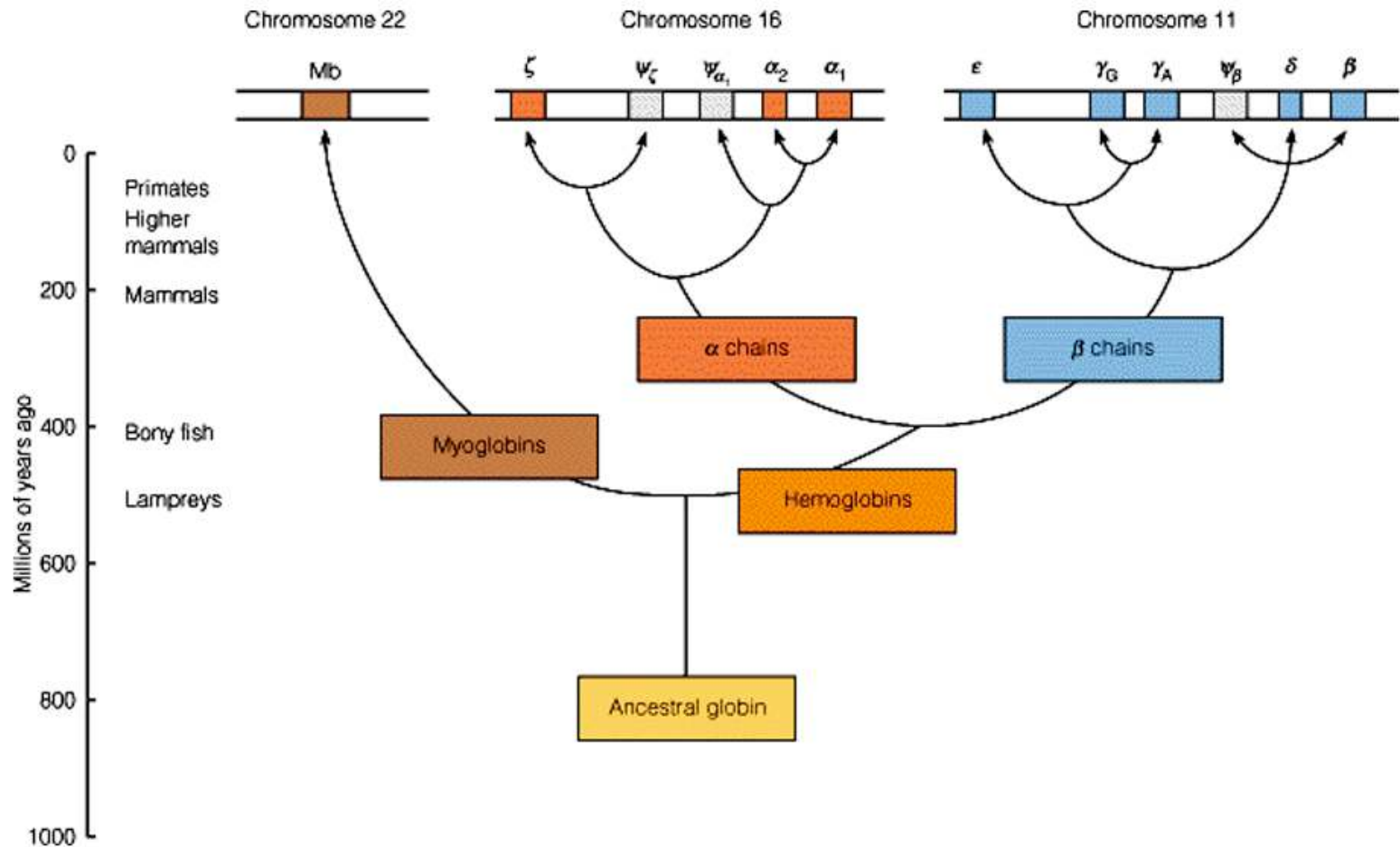
Sequence Alignment: Task Definition

- given:
 - a pair of sequences (DNA or protein)
 - a method for scoring a candidate alignment
- do:
 - determine the correspondences between substrings in the sequences such that the similarity score is maximized

Why Do Alignment?

- *homology*: similarity due to descent from a common ancestor
- often we can infer homology from similarity
- thus we can sometimes infer structure/function from sequence similarity

Homology Example: Evolution of the Globins



Homology

- homologous sequences can be divided into two groups
 - *orthologous sequences*: sequences that differ because they are found in different species (e.g. human α -globin and mouse α -globin)
 - *paralogous sequences*: sequences that differ because of a gene duplication event (e.g. human α -globin and human β -globin, various versions of both)

Issues in Sequence Alignment

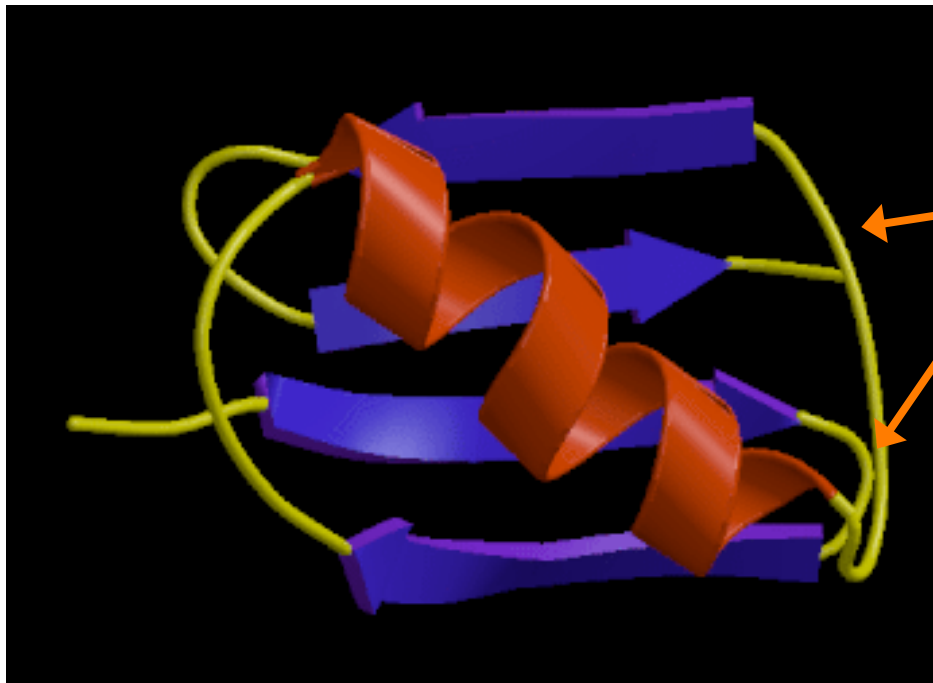
- the sequences we're comparing probably differ in length
- there may be only a relatively small region in the sequences that match
- we want to allow partial matches (i.e. some amino acid pairs are more substitutable than others)
- variable length regions may have been inserted/deleted from the common ancestral sequence

Sequence Variations

- sequences may have diverged from a common ancestor through various types of mutations:
 - substitutions (ACGA → AGGA)
 - insertions (ACGA → ACCGGAGA)
 - deletions (ACGGAGA → AGA)
- the latter two will result in *gaps* in alignments

Insertions, Deletions and Protein Structure

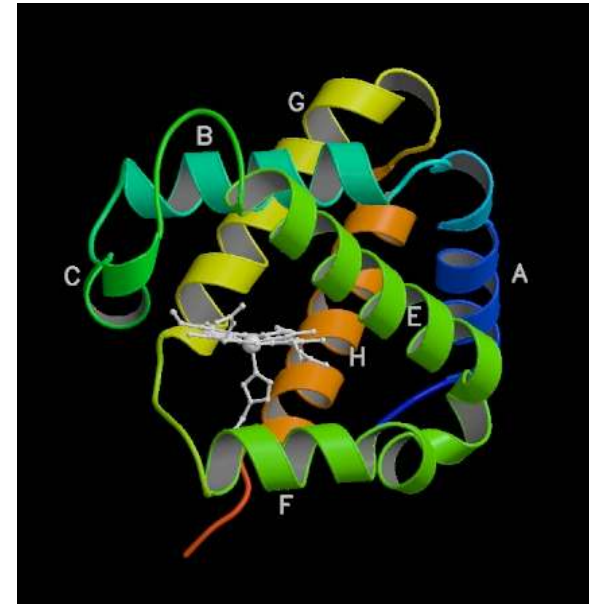
- Why is it that two “similar” sequences may have large insertions/deletions?
 - some insertions and deletions may not significantly affect the structure of a protein



loop structures: insertions/deletions here not so significant

Example Alignment: Globins

- figure at right shows prototypical structure of globins
- figure below shows part of alignment for 8 globins (-'s indicate gaps)



	A0	A4	A8	A17	B1	B6	B14	C2	CD1	CD4			
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓			
Hb_a	-----VL	SPADK	TNVKAA	WGKVGA	----	HAGEY	GAEAL	ERMFL	SFP	TTKTY	FPHF		
Hb_b	-----VHL	TPEEK	SAVTAL	WGKV	----	NVDEV	VGGEAL	GRLL	VVYP	WTQRF	ESF		
Mb_SW	-----VL	SEGEW	QLVLH	VWAKVEA	----	DVAGH	GQDIL	IRLF	FKSH	PETLEK	FDRF		
LegHb	-----GAL	TESQA	ALVKSS	WEEFNA	----	NIPKH	THRFF	FILV	LEIAPA	AKDLF	SFL		
BacHb	-----LDQ	QTINI	IKA	TVPLKEHG	----	V-T	TTTTF	YKNLF	AKHP	EVRLP	LF---		
SeaHb	GGTL	AIQA	QGDL	TLAOK	KIVRK	TWHOLMR	----	NKTS	FVTDV	FIRIF	AYDP	SAQNKF	PQM
AscHb	-----	ANK	TRELC	MKSL	ENAKVD	TSNEARQ	DGIDLY	KHMF	ENYP	PLRKY	FKS-		
Eryt.	-----	L	SADQ	I	STVQAS	FDKVKG	----	DPV	GILYA	VFKADP	SIMAK	F	TQF

Three Key Questions

- Q1: what do we want to align?
- Q2: how do we “score” an alignment?
- Q3: how do we find the “best” alignment?

Q1: What Do We Want to Align?

- *global alignment*: find best match of both sequences in their entirety
- *local alignment*: find best subsequence match
- *semi-global alignment*: find best match without penalizing gaps on the ends of the alignment

The Space of Global Alignments

- some possible global alignments for **ELV** and **VIS**

ELV
VIS

-ELV
VIS-

--ELV
VIS--

ELV-
-VIS

E-LV
VIS-

ELV--
--VIS

EL-V
-VIS

Q2: How Do We Score Alignments?

- gap penalty function
 - $w(k)$ indicates cost of a gap of length k
- substitution matrix
 - $s(a,b)$ indicates score of aligning character a with character b

Linear Gap Penalty Function

- different gap penalty functions require somewhat different dynamic programming algorithms
- the simplest case is when a linear gap function is used

$$w(k) = g \times k$$

where g is a constant

- we'll start by considering this case

Scoring an Alignment

- the score of an alignment is the sum of the scores for pairs of aligned characters plus the scores for gaps
- example: given the following alignment

VAHV---D--DMPNALSALSDLHAHKL
AIQLQVTGVVVTDATLKNLGSVHVSKG

- we would score it by
 $s(V,A) + s(A,I) + s(H,Q) + s(V,L) + 3g + s(D,G) + 2g \dots$

Q3: How Do We Find the Best Alignment?

- simple approach: compute & score all possible alignments
- but there are

$$\binom{2n}{n} = \frac{(2n)!}{(n!)^2} \approx \frac{2^{2n}}{\sqrt{\pi n}}$$

possible global alignments for 2 sequences of length n

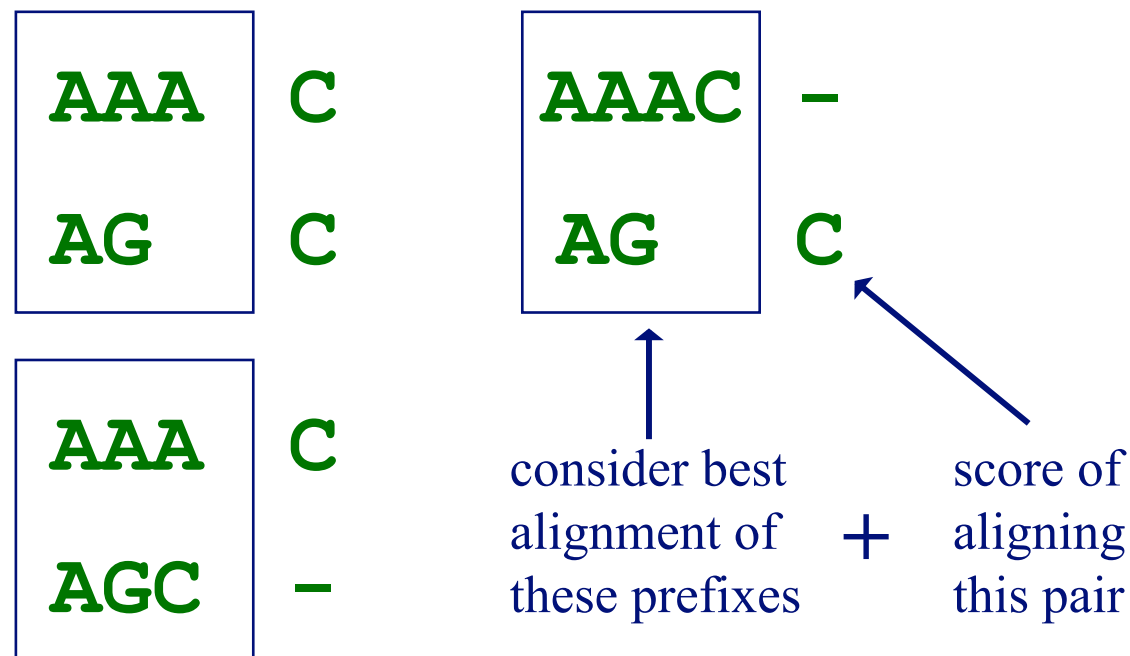
- e.g. two sequences of length 100 have $\approx 10^{77}$ possible alignments

Pairwise Alignment Via Dynamic Programming

- *dynamic programming*: solve an instance of a problem by taking advantage of solutions for subparts of the problem
 - reduce problem of best alignment of two sequences to best alignment of all prefixes of the sequences
 - avoid recalculating the scores already considered
 - example: Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34...
- first used in alignment by Needleman & Wunsch, *Journal of Molecular Biology*, 1970

Dynamic Programming Idea

- consider last step in computing alignment of **AAAC** with **AGC**
- three possible options; in each we'll choose a different pairing for end of alignment, and add this to best alignment of previous characters



Dynamic Programming Idea

- given an n -character sequence x , and an m -character sequence y
- construct an $(n+1) \times (m+1)$ matrix F
- $F(i, j) = \text{score of the best alignment of } x[1..i] \text{ with } y[1..j]$

	A	G	C
A			
A			
A			
C			

score of best alignment of AAA to AG

Needleman-Wunch Algorithm

- one way to specify the DP is in terms of its recurrence relation:

$$F(i, j) = \max \begin{cases} F(i-1, j-1) + s(x_i, y_j) \\ F(i-1, j) + g \\ F(i, j-1) + g \end{cases}$$

match x_i with y_j → $F(i-1, j-1) + s(x_i, y_j)$

→ $F(i-1, j) + g$ insertion in x

→ $F(i, j-1) + g$ insertion in y

DP Algorithm Sketch: Global Alignment

- initialize first row and column of matrix
- fill in rest of matrix from top to bottom, left to right
- for each $F(i, j)$, save pointer(s) to cell(s) that resulted in best score
- $F(m, n)$ holds the optimal alignment score; trace pointers back from $F(m, n)$ to $F(0, 0)$ to recover alignment

Initializing Matrix

	A	G	C
	0 ← g ← $2g$ ← $3g$		
A	g		
A	$2g$		
A	$3g$		
C	$4g$		

Global Alignment Example

- suppose we choose the following scoring scheme:

$$s(x_i, y_i) =$$

$$+1 \quad \text{when } x_i = y_i$$

$$-1 \quad \text{when } x_i \neq y_i$$

$$g \text{ (penalty for aligning with a gap)} = -2$$

Global Alignment Example

	A	G	C
A			
A			
A			
C			

$$s(x_i, y_i) =$$

+1 when $x_i = y_i$
-1 when $x_i \neq y_i$
 $g = -2$

Global Alignment Example

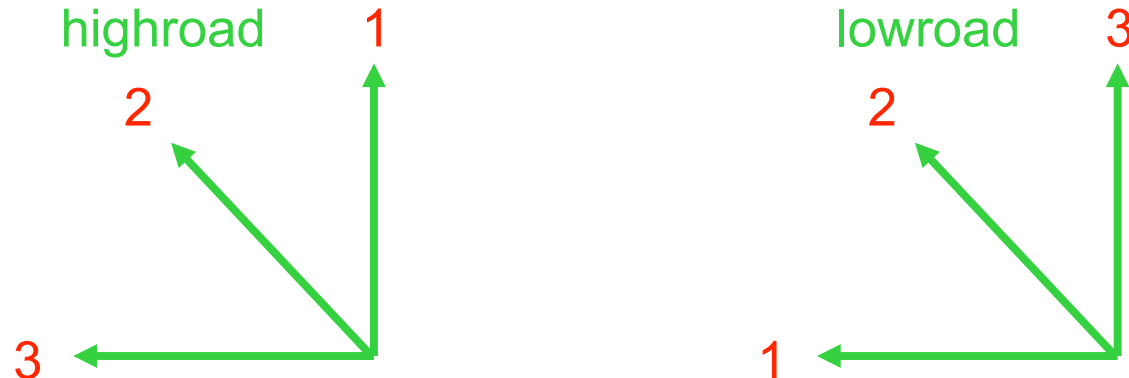
	A	G	C	
A	0	-2	-4	-6
A	-2	1	-1	-3
A	-4	-1	0	-2
A	-6	-3	-2	-1
C	-8	-5	-4	-1

one optimal alignment

x: A A A C
y: A G - C

Equally Optimal Alignments

- many optimal alignments may exist for a given pair of sequences
- can use preference ordering over paths when doing traceback



- *highroad* and *lowroad* alignments show the two most different optimal alignments

Highroad & Lowroad Alignments

	A	G	C
A	0 ← -2 ↑ -2	-2 ← -4 ↑ 1	-4 ← -6 ↑ -1
A	-2 ← -4 ↑ -6	1 ← -1 ↑ -3	-1 ← -3 ↑ -2
A	-4 ← -6 ↑ -8	-1 ← -3 ↑ -5	0 ← -2 ↑ -4
C	-6 ← -8 ↑ -1	-3 ← -5 ↑ -4	-2 ← -1 ↑ -1

highroad alignment

x: A A A C
y: A G - C

lowroad alignment

x: A A A C
y: - A G C

DP Comments

- works for either DNA or protein sequences, although the substitution matrices used differ
- finds an optimal alignment
- the exact algorithm (and computational complexity) depends on gap penalty function (we'll come back to this)

Local Alignment

- so far we have discussed *global alignment*, where we are looking for best match between sequences from one end to the other
- more commonly, we will want a *local alignment*, the best match between subsequences of x and y

Local Alignment Motivation

- useful for comparing protein sequences that share a common *motif* (conserved pattern) or *domain* (independently folded unit) but differ elsewhere
- useful for comparing DNA sequences that share a similar *motif* but differ elsewhere
- useful for comparing protein sequences against *genomic DNA sequences* (long stretches of uncharacterized sequence)
- more sensitive when comparing highly diverged sequences

Local Alignment DP Algorithm

- original formulation: Smith & Waterman, *Journal of Molecular Biology*, 1981
- interpretation of array values is somewhat different
 - $F(i, j)$ = score of the best alignment of a suffix of $x[1 \dots i]$ and a suffix of $y[1 \dots j]$

Local Alignment DP Algorithm

- the recurrence relation is slightly different than for global algorithm

$$F(i, j) = \max \begin{cases} F(i-1, j-1) + s(x_i, y_j) \\ F(i-1, j) + g \\ F(i, j-1) + g \\ 0 \end{cases}$$


Local Alignment DP Algorithm

- initialization: first row and first column initialized with 0's
- traceback:
 - find maximum value of $F(i, j)$; can be anywhere in matrix
 - stop when we get to a cell with value 0

Local Alignment Example

$$s(x_i, y_i) =$$

+1 when $x_i = y_i$

-1 when $x_i \neq y_i$

$$g = -2$$

T

T

A

A

G

A

A

G

A

More On Gap Penalty Functions

- a gap of length k is more probable than k gaps of length 1
 - a gap may be due to a single mutational event that inserted/deleted a stretch of characters
 - separated gaps are probably due to distinct mutational events
- a linear gap penalty function treats these cases the same
- it is more common to use an *affine* gap penalty function, which involves two terms:
 - a penalty h associated with opening a gap
 - a smaller penalty g for extending the gap

Gap Penalty Functions

- linear

$$w(k) = gk$$

- affine

$$w(k) = \begin{cases} h + gk, & k \geq 1 \\ 0, & k = 0 \end{cases}$$

Dynamic Programming for the Affine Gap Penalty Case

- to do in $O(n^2)$ time, need 3 matrices instead of 1

$M(i, j)$ best score given that $x[i]$ is aligned to $y[j]$

$I_x(i, j)$ best score given that $x[i]$ is aligned to a gap

$I_y(i, j)$ best score given that $y[j]$ is aligned to a gap

Global Alignment DP for the Affine Gap Penalty Case

$$M(i, j) = \max \begin{cases} M(i-1, j-1) + s(x_i, y_j) & \text{match } x_i \text{ with } y_j \\ I_x(i-1, j-1) + s(x_i, y_j) & \text{insertion in } x \\ I_y(i-1, j-1) + s(x_i, y_j) & \text{insertion in } y \end{cases}$$

$$I_x(i, j) = \max \begin{cases} M(i-1, j) + h + g & \text{open gap in } x \\ I_x(i-1, j) + g & \text{extend gap in } x \end{cases}$$

$$I_y(i, j) = \max \begin{cases} M(i, j-1) + h + g & \text{open gap in } y \\ I_y(i, j-1) + g & \text{extend gap in } y \end{cases}$$

Global Alignment DP for the Affine Gap Penalty Case

- initialization

$$M(0,0) = 0$$

$$I_x(i,0) = h + g \times i$$

$$I_y(0,j) = h + g \times j$$

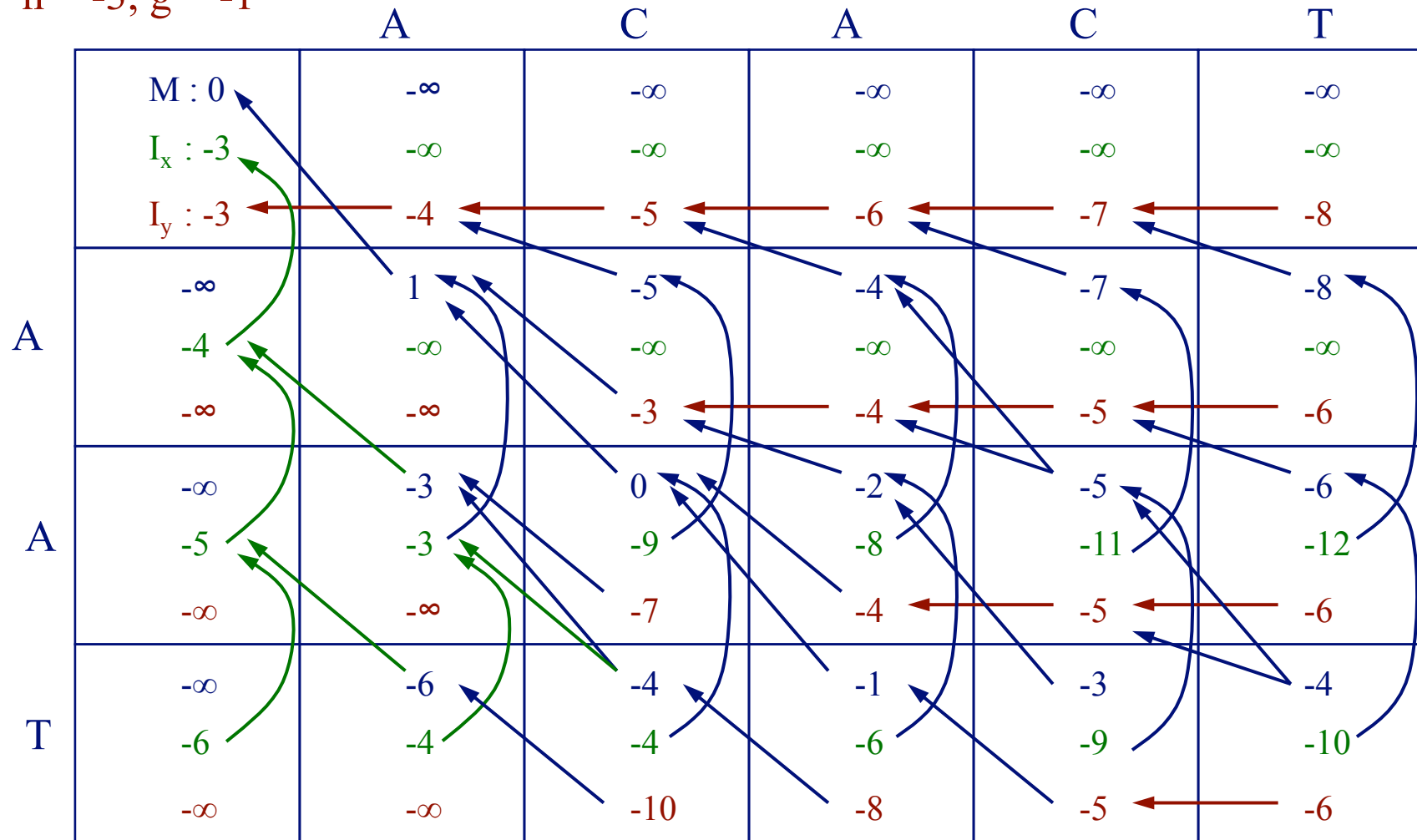
other cells in top row and leftmost column = $-\infty$

- traceback

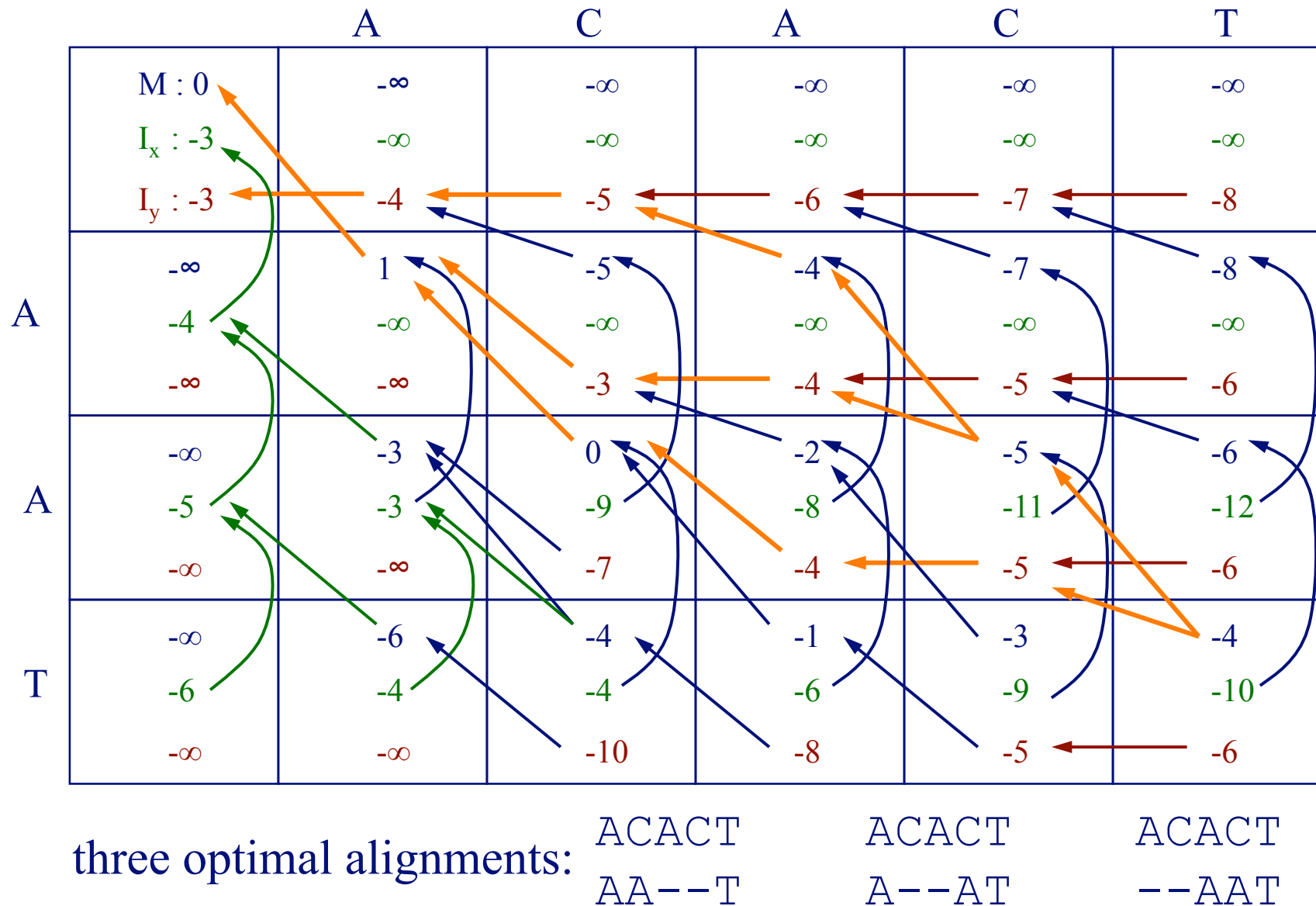
- start at largest of $M(m,n), I_x(m,n), I_y(m,n)$
- stop at any of $M(0,0), I_x(0,0), I_y(0,0)$
- note that pointers may traverse all three matrices

Global Alignment Example (Affine Gap Penalty)

$h = -3, g = -1$



Global Alignment Example (Continued)



Local Alignment DP for the Affine Gap Penalty Case

$$M(i, j) = \max \begin{cases} M(i-1, j-1) + s(x_i, y_j) \\ I_x(i-1, j-1) + s(x_i, y_j) \\ I_y(i-1, j-1) + s(x_i, y_j) \\ 0 \end{cases}$$


$$I_x(i, j) = \max \begin{cases} M(i-1, j) + h + g \\ I_x(i-1, j) + g \end{cases}$$

$$I_y(i, j) = \max \begin{cases} M(i, j-1) + h + g \\ I_y(i, j-1) + g \end{cases}$$

Local Alignment DP for the Affine Gap Penalty Case

- initialization

$$M(0,0) = 0$$

$$M(i,0) = 0$$

$$M(0,j) = 0$$

cells in top row and leftmost column of $I_x, I_y = -\infty$

- traceback

- start at largest $M(i,j)$

- stop at $M(i,j) = 0$

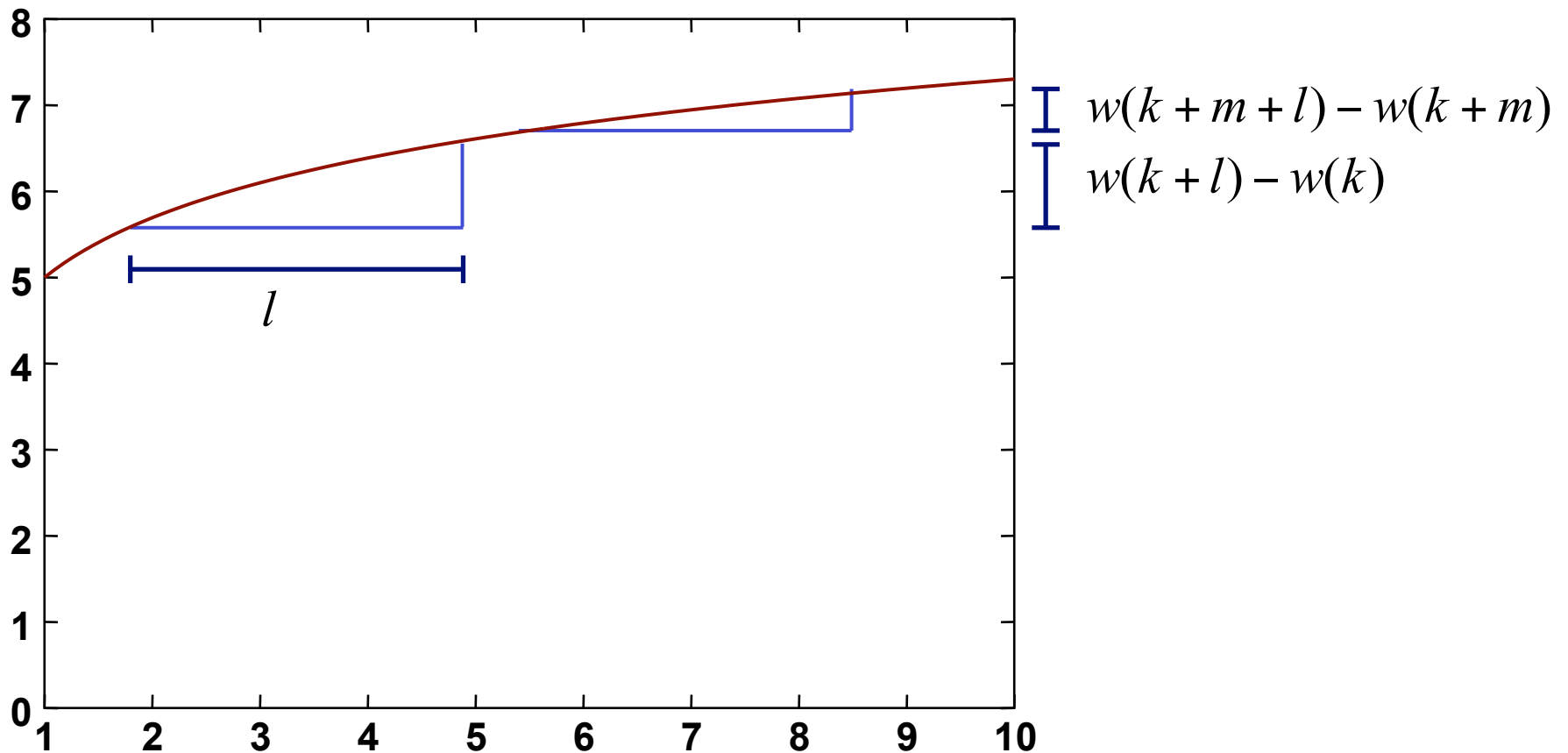
Gap Penalty Functions

- linear: $w(k) = gk$
- affine:
$$w(k) = \begin{cases} h + gk, & k \geq 1 \\ 0, & k = 0 \end{cases}$$
- concave: a function for which the following holds for all $k, l, m \geq 0$

$$w(k + m + l) - w(k + m) \leq w(k + l) - w(k)$$

e.g. $w(k) = h + g \times \log(k)$

Concave Gap Penalty Functions



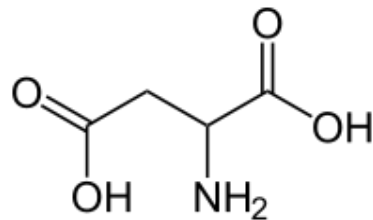
$$w(k + m + l) - w(k + m) \leq w(k + l) - w(k)$$

More On Scoring Matches

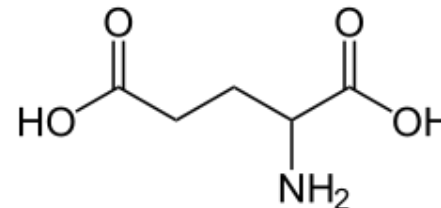
- so far, we've discussed multiple gap penalty functions, but only one match-scoring scheme:

$$s(x_i, y_i) = \begin{array}{ll} +1 & \text{when } x_i = y_i \\ -1 & \text{when } x_i \neq y_i \end{array}$$

- for protein sequence alignment, some amino acids have similar structures and can be substituted in nature:



aspartic acid (D)



glutamic acid (E)

Substitution Matrices

- two popular sets of matrices for protein sequences
 - PAM matrices [Dayhoff *et al.*, 1978]
 - BLOSUM matrices [Henikoff & Henikoff, 1992]
- both try to capture the the relative substitutability of amino acid pairs in the context of evolution

BLOSUM62 Matrix

BLOSUM62

A	4																			
R	-1	5																		
N	-2	0	6																	
D	-2	-2	1	6																
C	0	-3	-3	-3	9															
Q	-1	1	0	0	-3	5														
E	-1	0	0	2	-4	2	5													
G	0	-2	0	-1	-3	-2	-2	6												
H	-2	0	1	-1	-3	0	0	-2	8											
I	-1	-3	-3	-3	-1	-3	-3	-4	-3	4										
L	-1	-2	-3	-4	-1	-2	-3	-4	-3	2	4									
K	-1	2	0	-1	-3	1	1	-2	-1	-3	-2	5								
M	-1	-1	-2	-3	-1	0	-2	-3	-2	1	2	-1	5							
F	-2	-3	-3	-3	-2	-3	-3	-3	-1	0	0	-3	0	6						
P	-1	-2	-2	-1	-3	-1	-1	-2	-2	-3	-3	-1	-2	-4	7					
S	1	-1	1	0	-1	0	0	0	-1	-2	-2	0	-1	-2	-1	4				
T	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	1	5			
W	-3	-3	-4	-4	-2	-2	-3	-2	-2	-3	-2	-3	-1	1	-4	-3	-2	11		
Y	-2	-2	-2	-3	-2	-1	-2	-3	2	-1	-1	-2	-1	3	-3	-2	-2	2	7	
V	0	-3	-3	-3	-1	-2	-2	-3	-3	3	1	-2	1	-1	-2	-2	0	-3	-1	4
X	0	-1	-1	-1	-2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2	0	0	-2	-1	-1
	A	R	N	D	C	Q	E	G	H	I	L	K	M	F	P	S	T	W	Y	V

Positive for chemically similar substitution

Common amino acids have low weights

Rare amino acids have high weights

Heuristic Methods

- the algorithms we learned today take $O(nm)$ time to align sequences, which is too slow for searching large databases
 - imagine an internet search engine, but where queries and results are protein sequences
- heuristic methods do fast approximation to dynamic programming
 - example: BLAST [Altschul *et al.*, 1990; Altschul *et al.*, 1997]
 - break sequence into small (e.g. 3 base pair) “words”
 - scan database for word matches
 - extend all matches to seek high-scoring alignments
 - tradeoff: sensitivity for speed

Multiple Sequence Alignment

- we've only discussed aligning 2 sequences, but we may want to do more
- discover common motifs in a set of sequences (e.g. DNA sequences that bind the same protein)
- characterize a set of sequences (e.g. a protein family)
- much more complex

```

GGWWRGdy.ggkkqLWFP SN YV
IGWLNGyn.eftgerGDFPGTYV
PNWWEGql..nnrrGIFP SN YV
DEWWQAr r..deqqiGIVP SK -
GEWWKAqs..tgqqeGFIPFNFV
GDWWLAr s..sgqqtGYIP SN YV
GDWWDAel..kgrrrGKVP SN YL
-DWWEAr sls.sghrGYVP SN YV
GDWWYAr slitnseGYIP ST YV
GEWWKArs latrkeGYIP SN YV
GDWWLAr slvtgreGYVP SN FV
GEWWKAkslsskreGFIP SN YV
GEWCEAqt.kngq.GWVP SN YI
SDWWRVvnl.ttrqqeGLIPLNFV
LPWWRArd.kngqqeGYIP SN YI
RDWWEFrsktvytpGYIYESGYV
EHWWKVkd.algnvGYIP SN YV
IHWWRVqd.rnqheGYVP SS YL
KDWWKVev..ndrqqGFVPAAYV
VGWMPGlnerttrqrGDFPGTYV
PDWWEGel..ngqqrGVFP AS YV
ENWWNGei..gnrkGIFPATYV
EEWLEGec..kgkvGIFPKV FV
GGWWKGdy.gttriqqYFP SN YV
DGWWRGsy..ngqvGWFP SN YV
QGWWRGel..ygrvGWFP AN YV
GRWWKAr r..angetGIIP SN YV
GGWTQGel.ksgqkGWAPTNYL
GDWWEAr sn.tgenGYIP SN YV
NDWWTGrt..ngkeGIFP AN YV
    
```

Figure from A. Krogh, An Introduction to Hidden Markov Models for Biological Sequences

Next Time...

- basic molecular biology
- sequence alignment
- **probabilistic sequence models**
- gene expression analysis
- protein structure prediction
 - by Ameet Soni